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LETTER TO THE EDITOR

Dilution dependence of the relaxation time in the dilute Ising model

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Abstract. The magnetisation M in a three-dimensional Ising model, with only a fraction p of sites occupied randomly with spins, is found by Monte Carlo simulation to relax to zero logarithmically in time. The logarithm of the non-linear relaxation time $\int M(t)\,dt$ varies roughly as $1/(p_c-p)^{1/2}$ in the paramagnetic region.

The static properties of dilute magnets with well defined localised moments are much better understood (Stinchcombe 1983) than the dynamic properties of these systems. The dependence of the energy barriers on the size of the clusters has been investigated by Rammal and Benoit (1985) and Henley (1985) and may lead to unusual dynamics.

Recently we have investigated the dynamics of the random-field Ising model in three dimensions by Monte Carlo simulation (Stauffer *et al* 1984, Chowdhury and Stauffer 1985) using a multi-spin coding technique (see Kalle and Winkelmann 1982 and references therein). Following the basic principles of the latter algorithm, we have developed an algorithm for the simulation of simple cubic lattices with size up to 90^3 where a fraction p of the sites are occupied randomly by Ising spins. (We plan to publish the details of the algorithm elsewhere.) The Ising spins are assumed to interact mutually via nearest-neighbour ferromagnetic exchange interaction J with $J/kT_c = 0.221\,655$ for pure systems, $p=1$. Since the critical behaviour along the broken line shown in figure 1 is expected to have the same exponents as along the percolation line $T=0$, $p<p_c$ (Stinchcombe 1983), we have studied the former for convenience of Monte Carlo simulation which always requires non-zero temperature. The parameter $\exp(-2J/kT)$ has been taken as $1-p/p_c$. We have studied both the statistics as well as the dynamics of the model. However, in this letter we shall report only the data on the relaxation for $p<p_c$. The relaxation time τ is defined by

$$\tau = \int_0^\infty M(t)\,dt$$

where $M(t)$ is the magnetisation at time t, $M(\infty)=0$. In our simulation, all the pN ($N=90^3$) spins are assumed to be 'up' initially. Then, following the standard Metropolis algorithm we let the spin system evolve with time and we monitor the magnetisation at every Monte Carlo step (MCS). The simulation is terminated when M is fluctuating about zero; then ΣM_i is computed where i stands for the ith MCS

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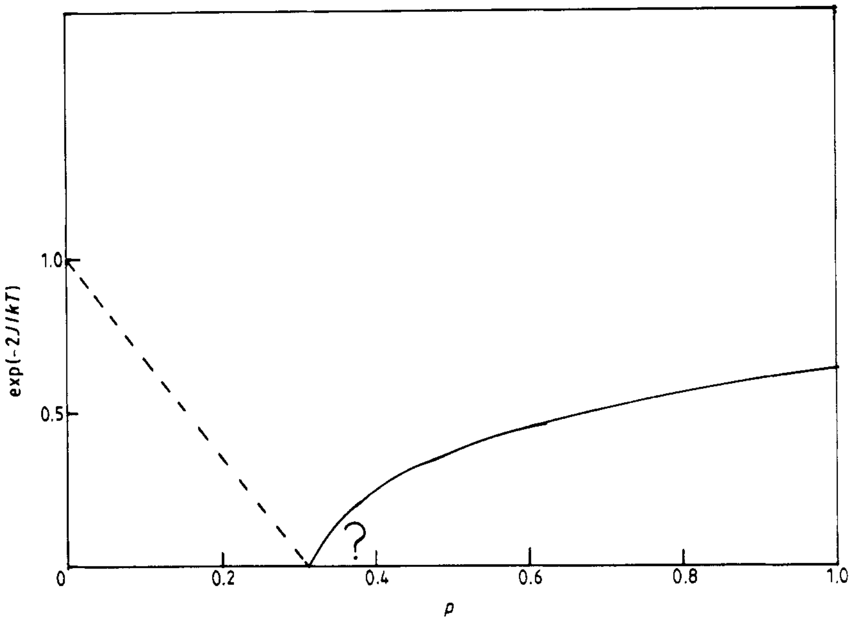

Figure 1. Phase diagram of dilute Ising model on simple cubic lattice. The curve to the right separates the ferro- and paramagnetic region. Our data were taken along the broken line to the left.

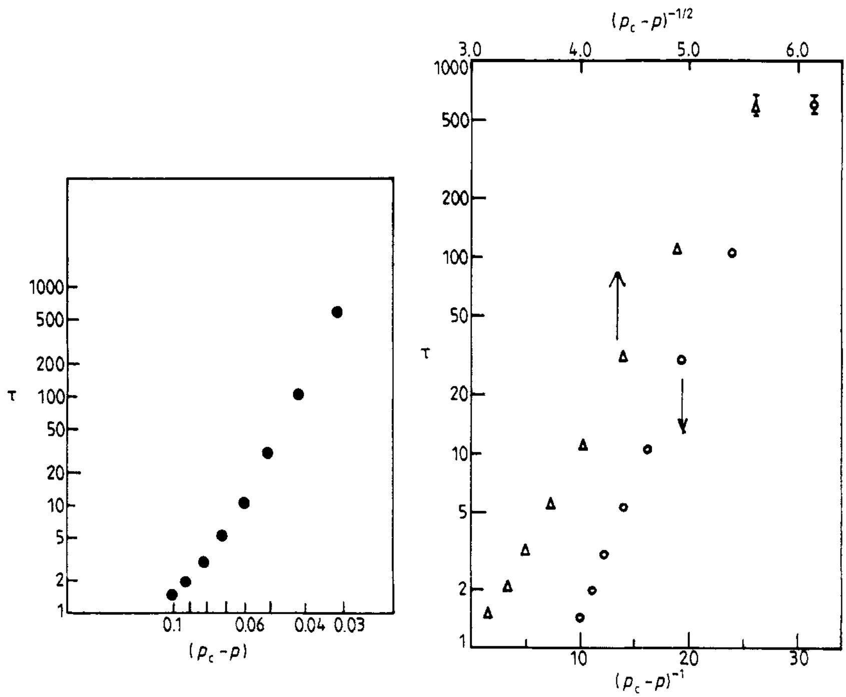

Figure 2. Variation of the relaxation time τ with concentration p. The left part is a log–log plot to check if a power law holds. In the right part we plot τ (logarithmic scale) against $1/(p_c-p)$ (circles, lower scale) and against $1/(p_c-p)^{1/2}$ (triangles, upper scale). The percolation threshold was taken as $p_c=0.3117$.

per spin, identified with the time $t = i$. The latter sum is the Monte Carlo estimate of $\tau(p)$.

We have plotted τ as a function of p in figure 2. The best straight line fit to the data is obtained for

$$\tau \propto \exp[\text{constant} \times (p_c - p)^{-1/2}].$$

However, the possibility of

$$\tau \propto \exp[\text{constant} \times (p_c - p)^{-1}]$$

or of a power law with a high exponent cannot be ruled out (see figure 2, where also a normal log-log plot is shown). Moreover, we have plotted the magnetisation as a function of time in figure 3. For $p > p_c$, the decay is certainly logarithmic over a large interval of time. Also for $p < p_c$ the decay seems to be logarithmic; however, the time interval of the validity of logarithmic decay is much smaller.

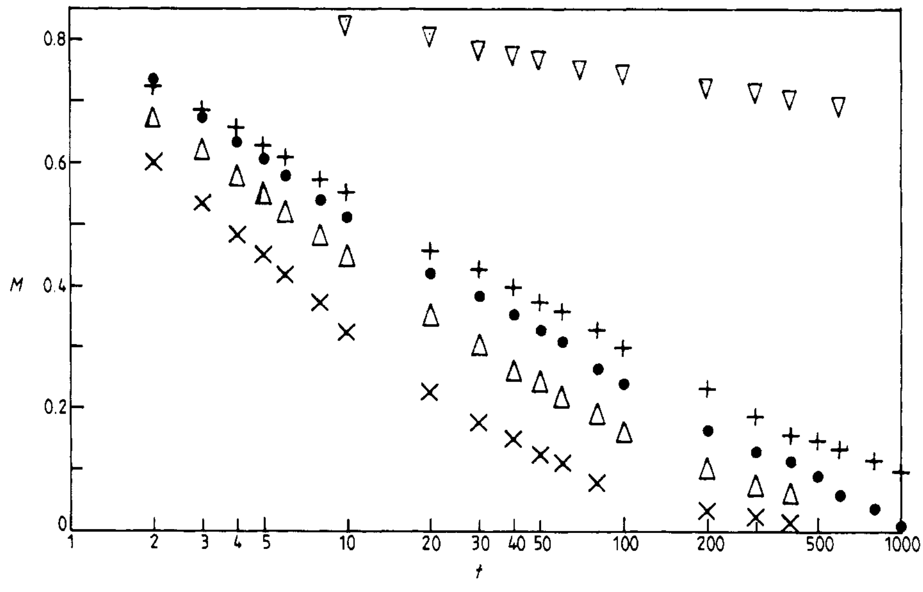

Figure 3. Relaxation of the magnetisation with time (logarithmic scale). The dots refer to $p = 0.95$ and $T/T_c(p=1) = 0.9513$, the crosses to $p = 0.26$ and $T/T_c(1) = 0.2468$, the triangles to $p = 0.27$ and $T/T_c(1) = 0.2204$, the plus signs to $p = 0.28$ and $T/T_c(1) = 0.1939$ and the inverted triangles to $p = 0.3067$ and $T/T_c(1) = 0.1073$.

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Note added in proof. Our data in the left part of figure 1 can be fitted well by a parabola, $\ln \tau = 1.66 \ln^2(p_c p) + 4.33 \ln(p_c - p) + 1.6$, as suggested by C K Harris and R B Stinchcombe (Preprint) (see also R Rammal 1985 *J. Physique* in press), similar to two dimensions (S Jain 1986 *J. Phys. A: Math. Gen.* **19** in press).

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